Fast

**National University of Computer & Emerging Sciences, Karachi  
Fall-2020 - Department of Computer Science**

**Bachelor of Science (Computer Science)  
Midterm 1 Examination -- Solution  
October 20, 2020, 01:15 pm – 02:15 pm**

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| **Course Code: CS211** | **Course Name: Discrete Structures** | |
| **Instructor Names: Dr. Fahad Samad, Mr. Shoaib Raza, Ms. Bakhtawer** | | |
| **Student Roll No:** | | **Section No:** |

**Instructions:**

* Return the question paper together with the answer script. Read each question completely before answering it. There are **3 questions and 2 pages.**
* In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
* Attempt all the questions in given sequence of the question paper.

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**Total Time**: 60 minutes **Maximum Marks**: 26

**Question # 1 (Propositional Logic and Rules of Inference)** **[5x2=10 points]**

(i) Let ***p*** and ***q*** be the propositions.

***p:*** Swimming at the New Jersey shore is allowed. ***q:*** Sharks have been spotted near the shore.

Write these propositions using ***p*** and ***q*** and logical connectives (including negations):

a) Swimming at the New Jersey shore is not allowed and either Swimming at the New Jersey shore is allowed

or sharks have not been spotted near the shore.

**Solution: ￢p ∧ (p V ￢q)**

b) Swimming at the New Jersey shore is allowed iff sharks have not been spotted near the shore.

**Solution: p ↔ ￢q**

(ii) Prove the following logical equivalence using the laws of logic:

***￢ [c ∨ (b ∧ (￢c →￢a))] ≅ ￢c ∧ (a ∨ ￢b)***

**Solution:**

***≡ ￢ c ∧ ￢* (*b ∧* (*￢ c → ￢ a*)) De Morgan’s law**

***≡ ￢ c ∧ ￢* (*b ∧* (*￢ ￢ c ∨ ￢ a*)) conditional rewritten as disjunction**

***≡ ￢ c ∧ ￢* (*b ∧* (*c ∨ ￢ a*)) double negation law**

***≡ ￢ c ∧* (*￢ b ∨ ￢* (*c ∨ ￢ a*)) De Morgan’s law**

***≡ ￢ c ∧* (*￢ b ∨* (*￢ c ∧ a*)) De Morgan’s law and double negation**

***≡* (*￢ c ∧￢ b*) *∨* (*￢ c ∧* (*￢ c ∧ a*)) distributive law**

***≡* (*￢ c ∧￢ b*) *∨* ((*￢ c ∧ ￢ c*) *∧ a*) associative law**

***≡* (*￢ c ∧￢ b*) *∨* (*￢ c*** ***∧ a*) idempotent law**

***≡ ￢ c ∧* (*￢ b ∨ a*) distributive law**

***≡ ￢ c ∧* (*a ∨ ￢ b*) commutative law**

**Hence Proved.**

(iii) Determine using truth table that whether the following is a tautology, contradiction or a contingency.

***[(p → q) ∧ (q → r)] → (p → r)***

**Solution:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **P** | **q** | **r** | **p → q** | **(q → r)** | **(p → q) ∧ (q → r)** | **(p → r)** | **[(p → q) ∧ (q → r)] → (p → r)** |
|  | | | | | | | |
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| T | F | F | F | T | F | F | T |
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| F | T | F | T | F | F | T | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |

**Hence, it’s a Tautology.**

(iv) What relevant conclusion or conclusions can be drawn from the following premises? Also, explain the rules of inference used to obtain each conclusion from the premises.

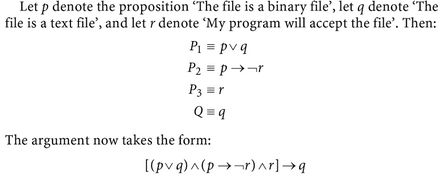
“The file is either a binary file or a text file.” “My program won’t accept the file if it’s a binary file.”

“My program will accept the file.”

Assume,

***p =*** “The file is a binary file.” ***q =*** “The file is a text file.”  ***r =*** “My program will accept the file.”

**Solution:**



(v) Write the negation of the following sentences in English.

a) If Jaffar lives in Pakistan, then he lives in Karachi.

**Solution: Jaffer lives in Pakistan and he does not live in Karachi (p*∧ ￢* q).**

b) If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

**Solution: n is divisible by 6 but n is not divisible by 2 or by 3 (p *∧ ￢* (q v r)).**

**Question # 2 (Predicate and Quantifiers) [3x2=6 points]**

(i) Let ***F (a, b)***means***“a +3b = ab****”,* where a and b are Positive integers. Determine the truth value of the statement.

a) *∀a ∃ b F (a, b).* **Solution: False**

b)  *∀ b ∃ a ¬F (a, b).* **Solution: True**

(ii) Let ***B (x)*** be the statement **"x has an Internet connection"** and ***C (x, y)*** be the statement **"x and y have chatted over the Internet,"** where the domain for the variables ***x*** and ***y*** consists of all students in your class.

Write the statement in good English without using variables in your answers.

*a) ¬*∀*x B (x).* **Solution: Not everyone in your class has an Internet connection.**

b) *∃x B(x) ∧ ∀y ¬C (x, y)*

**Solution: Someone in your class has an Internet connection but has not chatted with anyone else in your class.**

(iii) Express the following sentences using logical expression with nested quantifiers:

a) The Sum of two negative integers numbers is negative.

**Solution: ∀x ∀y ((x < 0) ∧ (y < 0) → (x + y < 0))**

b) The difference of two positive integers is not necessarily positive.

**Solution: *¬*∀x ∀y ((x > 0) ∧ (y > 0) → (x - y > 0))**

**Question # 3 (Functions and Set theory) [5x2=10 points]**

(i) Let ***f* : R → R** be defined by the formula ***f(x) = 4x-1 ∀x ∈ R.*** Is ***f*** a bijective function? If no, give reason why? If yes, find its inverse.

**Solution:**



(ii) Let ***f : Z →Z*** and ***g: Z → Z*** be defined by ***f(n) = n+1*** for ***n ∈ Z*** and ***g(n) = n2*** for ***n ∈ Z.***

a) Find the compositions ***gof*** and ***fog***.

b) Is ***gof = fog*** ?

**Solution:**

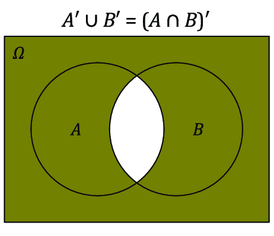
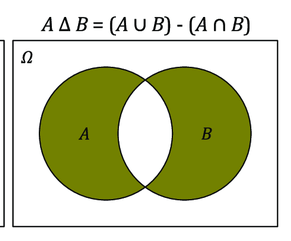


(iii) Draw Venn Diagram of the following relationships between the sets:

a) (A U B) – (A ∩ B)

b)  *U*

**Solution:**



(iv) Using set-builder notation, prove or disprove the following set operations:

***A − (B ∩ C) = (A − B) ∩ (A – C).***

**Solution:**

*A − (B ∩ C) = (A − B) ∩ (A − C)*

*A*∩ = (*A*∩ (*A*∩ A∩ = (A − B)

*A*∩ ∪ = (*A*∩A) ∩ De-Morgan and Associative Law

*A*∩ ∪ ≠ *A* ∩ Idempotent Law

Hence, it a disproof.

(v) Among a group of 165 students, 8 are taking calculus, psychology, and computer science; 33 are taking calculus and computer science; 20 are taking calculus and psychology; 24 are taking psychology and computer science; 79 are taking calculus; 83 are taking psychology; and 63 are taking computer science. How many are taking none of the three subjects?

**Solution:**





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***ALL THE BEST😊***